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THE STRENGTH OF THE EARTH'S CRUST

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PART VII. VARIATION OF STRENGTH WITH DEPTH AS SHOWN BY THE NATURE OF DEPARTURES FROM ISOSTASY

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INTRODUCTION AND SUMMARY

The first five parts of this article have concurred in showing that the crust is very strong when measured by its capacity to support great deltas, individual mountain ranges, or great internal loads due to irregularities in density not in accord with the topography. On the other hand, the altitudes of the continents as a whole, or of large sections of the continents, agree with the demands of nearly perfect isostasy. In Part VI it was shown, however, that, although even perfect isostasy threw very considerable stresses

[1] Section B, on Applications of the Theory, will be published in the following number of this *Journal*. The Introduction and Summary apply however to both sections.

upon the outer part of the crust, the maintenance or restoration of the isostatic condition through geologic time in spite of the opposing geologic activities implied the existence of an undertow below the zone of compensation. The existence of this regional isostasy for continental interiors as well as for ocean basins suggests, furthermore, that this zone of undertow is both thick and relatively very weak to resist shearing stresses. But if such a zone exists it must have important bearings on other branches of terrestrial dynamics besides that of isostasy. Its importance gives it a right to a distinct name, and it has been called here the zone of weakness—the asthenosphere. It is desirable to test its reality and its character by other lines of evidence, and such another line forms the basis of this part.

George H. Darwin has investigated the problem of the stress-differences imposed on the earth by the weights of continents and mountains. In his work the earth was assumed to possess competent elasticity throughout and the topography to be without isostatic compensation. Love has more recently treated the contrary problem of the isostatic support of continents and mountains, assuming as governing conditions that isostatic compensation was perfect within a depth of one-fiftieth of the earth's radius, 127 km., and that all shearing stresses due to topography and compensation disappeared at that depth. Below there is assumed to exist only hydrostatic pressures. In other words, Darwin postulates no sostasy and no asthenosphere; Love postulates perfect isostasy and a perfect asthenosphere. As there is known to be no truly fluid universal shell within the earth, and as isostasy for limited regions is far from perfect, the truth must lie between these two extremes. The asthenosphere must have some degree of strength and a measure of its strength is derived in this part by a study of the nature of the departures from isostasy.

For this purpose is discussed the nature of the stresses as worked out by Darwin. Then the departures from isostasy are analyzed into harmonic series. Those of long wave-length are seen to be of low amplitude, those of short wave-length of high amplitude. Now the departures from isostasy are according to their very nature without compensation and their stress effects will therefore

follow Darwin's law except in so far as the great rigidity of the outer crust will permit it to sustain loads after the manner of a continuous beam. It is shown, however, that the outer crust is inefficient as a beam, so that the results of applying Darwin's analysis will probably not be greatly modified.

It is found that the departures from isostasy are such as throw great stress-differences upon the zone of compensation, here called the lithosphere. The maximum stresses imposed by the loads found to exist within the United States lie furthermore within the outer two-thirds of that zone. The stress-differences due to this cause reach maxima probably between 3,000 and 5,000 pounds per square inch.

Within the asthenosphere, on the contrary, the stresses caused by the departures from isostasy are very small, under the United States the stress-differences at depths of from 400 to 600 km. reaching maxima probably between 500 and 600 pounds per square inch, between a sixth and tenth of those existing at higher levels.

Now the nature of those geologic actions which oppose isostasy, both the great compressive movements and the great cycles of erosion and sedimentation, are such that they tend to destroy the isostatic adjustments of whole continents and large parts of continents. By these broad actions they tend to bring larger and larger stress-differences upon the zone lying more than 200 km. deep. The limitation of their action as shown by the dominance of regional isostasy is therefore to be regarded as an effect of weakness in that zone. This then is another proof of the reality of an asthenosphere. The proof in Part VI depended upon the dynamics necessary for isostatic undertow; the proof in this part depends upon the limitations of stress with depth as measured by the existing departures from isostasy.

This is as far as the present fragmentary data and imperfect theory can safely go, but in order to visualize the arguments a curve of strength is given which shows how great a falling-off of strength there is from the upper part of the lithosphere to the middle of the asthenosphere. Below, the strength undoubtedly again increases, but the evidence for that is supplied by other lines of research than that opened by the geodetic data.

The results of this part suggest that in future investigations by mathematicians upon the elastic competence of the earth, a probable case would be to consider the isostatic compensation as uniformly tapering out through a depth twice as great as the depth given for uniformly distributed compensation, that is, tapering out through about 244–254 km. Further, it is not in accordance with nature to assume that at this depth all shearing stresses disappear. Such an assumption brings in artificialities nearly as great as those involved in Darwin's assumption of no isostatic compensation. Rather should the stress relations be solved as limited by some such curve as is here shown, and determined by the nature of the departures from isostasy. It is possible that this may add still further difficulties to the mathematical treatment of the subject, yet only by closer recognition of the realities of nature can mathematical analysis become of increasing value.

SECTION A, PRESENTATION OF THEORY

RELATIONS OF LOADS AND STRESSES

Stresses imposed by harmonic surface loads.—A harmonic series gives a succession of sweeping curves such as are shown in Fig. 15. The vertical scale may be made of any size and the curves may be regarded as sections across a series of hills and valleys, or, on progressively larger scales, anticlinoria and synclinoria, geanticlines and geosynclines, continents and ocean basins. The parts of the curves convex upward will then represent loads above the mean surface and give rise to stresses acting downward. The broad hollows give negative loads and the surface beneath is strained upward by the pressures from surrounding regions. The inequalities of the earth's surface may be taken as approximating to harmonic undulations of simple or complex nature. By so taking them, the stresses which they produce on the earth's interior may be evaluated.

G. H. Darwin treated this problem in his paper "On the Stresses Caused in the Interior of the Earth by the Weight of Continents and Mountains."[1] In this are investigated the stresses given by positive and negative loads whose distribution follows a law

[1] *Phil. Trans. Royal Soc.*, CLXXIII (1882), 187–230.

of zonal harmonics arranged on the surface of a sphere. The harmonic series of the second order corresponds to oblateness of a spheroid and also serves as a basis for computing the tidal strains. The zonal harmonics of the fourth order correspond to an equatorial continent and two polar continents, the eighth order adds to these two annular continents in about latitude 45°, and so on. The higher orders, above thirty, correspond to a succession of anticlinoria and synclinoria, or mountains and valleys. For all above the second harmonic the depth of maximum stress lies within the outer half of the earth's radius.

Darwin's solutions were made on the assumption that there are no differences of density beneath continents and oceans and that all the relief of the earth is upheld by its rigidity. He reached the conclusion that continents such as Africa and America gave a maximum stress-difference of about four tons per square inch at a depth of about 1,100 miles. The later demonstration of the existence of regional isostasy nullifies this conclusion except for the amount by which the topography of large areas is not completely compensated. Even this part can to some extent be regarded as sustained by a rigid crust floating upon a deeper zone which acts dynamically nearly as a fluid. There are reasons for believing, however, that this latter conception is extreme in the other direction and not justified by the evidence. It is thought that by the collective support of the arguments brought out in this part the assumption will be finally justified—that for the outstanding loads not in isostatic equilibrium the work of Darwin continues to apply.

In the mathematical analysis, the loads which represent the areas and heights of the regional departures from isostasy are regarded as members of an infinite harmonic series of ridges and furrows disposed on a plane. One wave-length is the distance from crest to crest, or mid-furrow to mid-furrow. Darwin showed, as illustrated in Figs. 15 and 16, that the magnitude of the stress-difference at any point within the crust due to a surficial harmonic load depended upon the depth below the mean horizontal surface measured in terms of the wave-length and not at all on the position of the point considered with reference to the ridges and furrows. Further in regard to the direction of the stress-difference, it is

shown, as illustrated in Fig. 15, that in passing uniformly and horizontally through the crust on a line at right angles to the direction of the ridges, the stress axes revolve with a uniform angular velocity. In relation to depth, the maximum stress-difference, as shown in Figs. 15 and 16, occurs at a depth equal to $\frac{1}{2\pi}$ of the wave-length and is then equal to $2\,gwh\epsilon^{-1}$ or in gravitation units of force to $0.736wh$, in which h is the height from the mean plane to the top or bottom of the undulations and w is the weight of a unit volume.

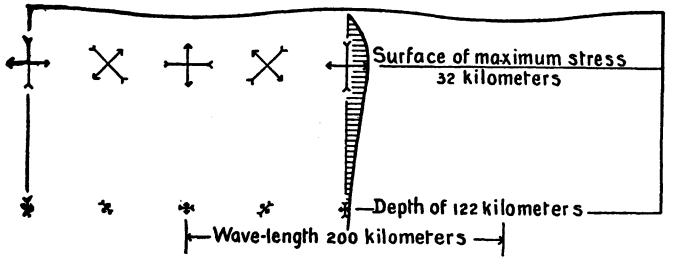


FIG. 15.—Diagram showing in vertical section uncompensated harmonic mountains and valleys with relative magnitude and direction of stress-differences, which they impose on the crust below. Mountain crests drawn as 5 km. above valley bottoms. Wave-length 200 km. Stresses shown to a depth of 122 km. Maximum stress for this wave-length is at 32 km.

It is important to note that the value of this maximum depends only on the height and density of the mountains and is independent of the distance from crest to crest. The depth at which this maximum is reached depends, on the other hand, upon the wave-length and not upon the height or density of the mountains. The effect of a doubling of the wave-length upon the vertical distribution of the stress-differences is shown in Fig. 16.

It is seen that the lateral pressure due to the elevations, instead of being at the surface as it would be under hydrostatic conditions or as in completely compensated mountains and valleys with the special distribution of density assumed by Love,[1] is at a depth of about one-sixth of the wave-length. The maximum stress is, furthermore, but 37 per cent of the full amount of the hydrostatic

[1] *Some Problems of Geodynamics* (1911), chaps. ii and iii.

lateral pressure. Fig. 16 shows that uncompensated features with a wave-length up to 200 km. impose the stresses almost wholly within the lithosphere, taking this as limited by a depth of 122 km.

The crust increases in strength to a certain maximum, perhaps from 10 to 30 km. deep, as shown by the experiments of Adams.

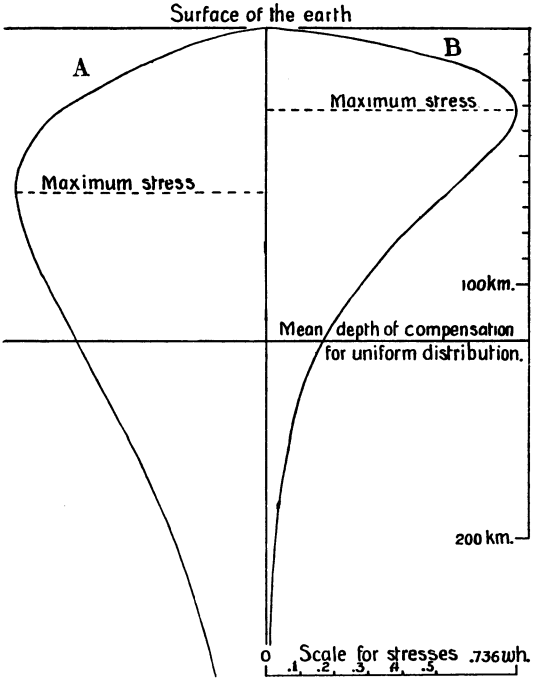


FIG. 16.—Diagram showing the distribution in the crust of stress-differences due to parallel uncompensated harmonic mountains and valleys: A, curve showing relative magnitude and depth of stress-differences corresponding to a distance of 400 km. between crests of parallel mountain ranges. B, curve showing the same for a distance of 200 km. between crests of parallel mountain ranges.

At greater depth the strength, according to the theory developed in Part VI, and as recognized by Love and others in treating of isostasy, is to be regarded as decreasing and passing by transition into the asthenosphere. Consequently it is seen that the distribution of stress imposed by a wave-length of 200 km. conforms well to the distribution of strength, the greatest strain coming on the strongest part.

Darwin obtained his results by making the initial assumption that the earth substance was incompressible but possessed elasticity of form. The introduction of the factor of compressibility Darwin showed to affect the result largely in the case of the second harmonic, but for harmonics of infinitely high orders the resulting stresses are independent of the modulus of compression. Consequently he states, "it may be concluded that except for the lower harmonic inequalities compressibility introduces but little change in our results."[1]

Modifications imposed by long and large wave-lengths.—In Fig. 16, curve A shows that for a wave-length of 400 km. the depth of maximum stress is 64 km. and at 122 km. the stress is 75 per cent of the maximum. If this should be regarded as the beginning of the asthenosphere it would mean that a large part of the stress would be thrown on to the zone incapable of sustaining large stress-differences. If the wave-length became 2,000 km. the lithosphere would be subjected to but small stress-differences and the maximum strain would occur at a depth of 320 km., the middle of the sphere of weakness. In order that Darwin's solution should hold for these cases the height of the arches would have to be so small that the resulting stress-differences would not exceed the elastic limit of the asthenosphere. For greater loads disposed on the surface in these large wave-lengths the stress relations would approach those of a rigid crust overlying a fluid substratum. Of this problem Darwin states, "The evaluation of stresses in a crust, with fluid beneath, would be tedious, but not more difficult than the present investigation" (on the stresses caused by the weight of continents and mountains).[2] It is a different problem from that solved by Love; the latter considering the stresses in such a crust caused by a condition of isostasy, not by a lack of isostasy. The limited mathematical training of the present writer does not permit here the definite solution of this problem, but some general observations can be made.

For wave-lengths very large in comparison with the depth of the lithosphere the stress-differences, if confined within the crust, approach those existing in a continuous beam, each span being

[1] *Scientific Papers*, II, 500. [2] *Ibid.*, II, 502, footnote.

stressed by a continuous load, greatest at the center in accordance with the harmonic curve and acting in the reverse direction from the adjacent spans. This is a very simple limiting case. The maximum bending moment would be on the cross-section at the crest of each downward and upward arch. For a given height of load the bending moment would increase with the square of the span or wave-length. The maximum bending stresses would be horizontal, acting as tensile and compressive stresses at the top and bottom of the lithosphere. In the middle of the lithosphere there would exist a neutral surface suffering neither tension nor compression, but subjected to horizontal shear. The theory of beams shows that the strength is limited by the marginal tensions and compressions, not by the internal shear. As the lithosphere is, however, weakest on the upper and lower margins, its material is poorly arranged to resist the bending stresses. The greatest resistance to bending in a certain plane is given by the form of an I-beam, but the crust is analogous to a beam in which a single flange should intersect its middle, giving a cross-shaped section. The earth's crust is consequently a peculiarly weak structure to resist harmonic loads of great wave-length, and as the strength varies inversely with the bending moment it varies inversely with the square of the wave-length. It is seen then that wave-lengths of continental breadth are very poorly supported by the strength of the crust, but if they reach notable amplitude must rest chiefly upon the asthenosphere. The consideration of the stress diagram given by a wave-length of 200 km. showed why very pronounced departures from isostasy can occur in one direction over areas up to at least 100 km. across and why marked regional compensation extends commonly to limits of 100–200 km. radius. The stress effects produced by harmonic loads a thousand kilometers or more in a wave-length show, on the other hand, why regional compensation of the same vertical magnitude cannot extend effectively across a whole continent. It explains why the United States as a whole is in nearly perfect isostatic equilibrium with respect to the ocean basins.

Nature of stresses imposed by internal loads.—Take the case of harmonic loads distributed on a plane *S–S*, Fig. 17, within an

indefinitely extended elastic solid. The amount and direction of the vertical stress upon this plane are shown by the vertical lines, the scale of stresses being one-twentieth of the value for the stress-diagram shown by the horizontal lines. On this plane, *S–S*, a small unit mass is subjected to stress equal in all directions and not to stress-difference, since the stress is essentially the same on the small contiguous unit masses. The reasoning is the same as

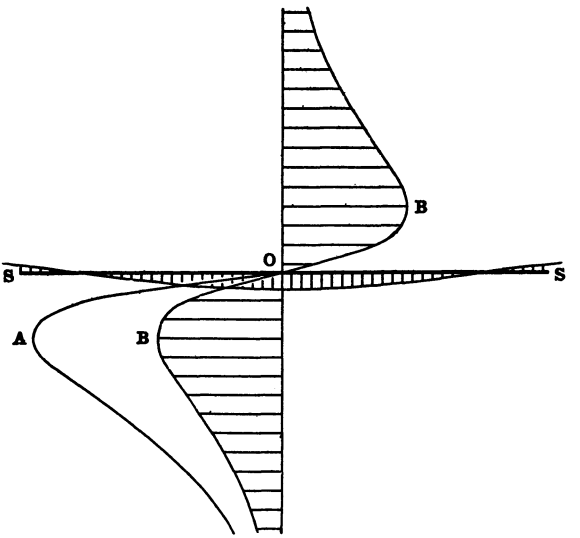


FIG. 17.—Stress-diagrams for harmonic loads distributed on a plane: A, diagram for loads upon a limiting surface of a solid extending indefinitely from this surface. B, diagram for loads upon a plane within an indefinitely extended solid. The scale for loads is one-twentieth of the scale for the resulting stresses. A little more than one-half wave-length is shown.

that for harmonic loads distributed on the limiting surface of a solid except that here there is an indefinitely extended solid on each side of the plane. The stress at any point of the plane acts positively on one side, negatively on the other. Half of the load will be carried on each side. Consequently if *OA* is the curve showing the stress-differences at various depths for a harmonic load on the surface of the earth, then *BOB* will be the stress-curve for the same load carried on a plane deep within the lithosphere. For this to be approximately true, however, the wave-length would have to be

small in comparison with the depth of the loaded surface. Then the upper half of the stress-diagram will lie within the lithosphere. The lower half of the curve would also have to lie within the elastic competence of the crust for corresponding depths. Suppose the plane to lie near to either boundary of the lithosphere. The case now approaches that of a surface load, one side of the stress-diagram becomes largely cut off and the other increases. The exact analysis is of course difficult and will not be attempted.

Assume the loaded plane to be at a depth of 61 km., half the depth of the lithosphere. The strength of the middle would not then be utilized for support of the load. For a wave-length up to 100 km. Fig. 17 shows fairly well the distribution of stress and it would be contained mostly within the middle of the lithosphere. For wave-lengths of 200 km., however, the margins of the lithosphere would be subjected to greater strain than the interior, the stress-diagram would be modified toward that existing in a loaded beam, and, if the margins are weak, the structure is poorly adapted to support the load. If the loaded plane is at greater depth, the same wave-length will throw a greater proportion of the strain upon the asthenosphere and the deeper parts of the lithosphere. If these are incapable of supporting the resulting stresses, again a modification of the diagram would occur, involving bending moments in the stronger part of the crust.

If, now, it be assumed that the upper half of the lithosphere is decidedly stronger than the lower half and that the maximum strength is at some depth below the surface, it is seen that the maximum outstanding masses which the crust could carry would be disposed in the outer quarter or third of the lithosphere. But this is just what was found to be the case as the result of the studies made in Part V. Therefore the accordance between the geodetic evidence and the consequences of the assumption raise it to the dignity of a presumption. It may be taken as a working hypothesis that the greater outstanding masses are limited in their positions by the limitations of crustal strength. Mental reservation must be made, however, as to the possibility of other more important determining factors, such for example as the nature of igneous activity, in limiting the zone of large outstanding masses. An

accordance of fact with theory is not a proof, but it raises a presumption that the theory is correct.

Nature of stresses imposed by perfect isostasy.—This topic, although not directly in line with the subject of this chapter, must receive brief mention here since the stresses in the crust are compounded of those due to the departures from isostasy with those resulting from a state of perfect isostasy. The stress resulting from the isostatic support of continents and mountains has been ably worked out by Love.[1] But his treatment started with the limiting though improbable assumption that at a depth of one-fiftieth of the earth's radius all stress-differences disappeared, as though the layer below were truly fluid. This required a complicated and equally improbable curve of density, opposite in sign above and below, in order that the topography should be compensated, the ocean surface remain a level surface, and yet the stress-differences become zero at the required depth. Nevertheless the solution is valuable as a limiting case in showing the general character of the internal stresses which must exist. He showed that isostatically compensated harmonic mountains and valleys gave maximum stress-differences on the axial lines of mountains and valleys and that it amounted to about one-fourth of the theoretical hydrostatic pressure. The stresses decrease rapidly with depth.

In contrast to Love's hypothesis of the distribution of density, that of Hayford may be considered. This is that the excess or defect in density needed for compensation is uniformly distributed to a depth of 122 km. Again, a rigorous mathematical treatment must be left to those competent to undertake it, but it would appear that such distributions of density would throw very considerable stress-differences within the asthenosphere; or, if this was incompetent to carry such, would bring large stress-differences upon the bottom of the zone of compensation, opposite in sign to that in the upper half, whereas at an intermediate level depending upon the wave-length would be a region of no stress-difference. This distribution of stress resulting from the hypothesis may be taken as a strong argument against the existence of a uniform distribution

[1] *Some Problems of Geodynamics* (1911).

to the isostatic compensation. In order to have the larger relief and its compensation fitted to a crust strongest in its upper part and shading into a zone of weakness, the zone of compensation should die out with depth and lie mostly in the upper half of the zone of strength, since the stress-differences would die out at a depth greater than the disappearance of the compensation. This conclusion is seen to be in closer accord with several other lines of evidence which have been noted than is the contrary assumption of uniform compensation.

[*To be continued*]